

Progression in Calculations at KS2

Understanding and Using Calculations

For all calculations, children need to:

- Understand the = sign as **is the same as** .
- See calculations where the equals sign is in different positions, e.g. $3 + 2 = 5$ and $5 = 7 - 2$.
- Approximate before calculating and check whether their answer is reasonable.

Addition – a definition

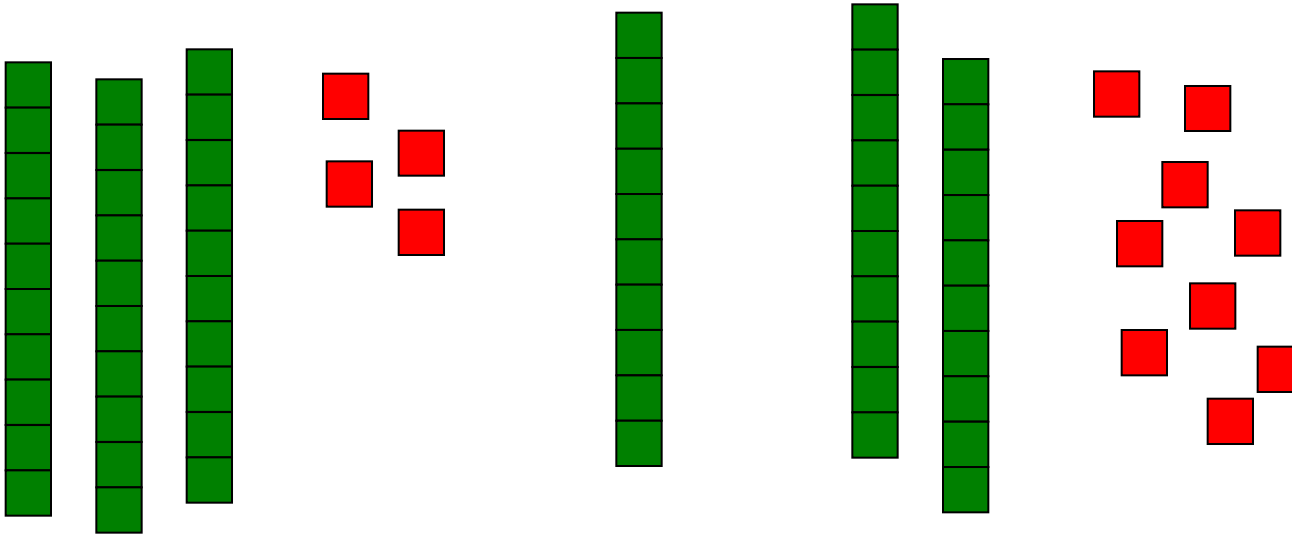
Children need to understand the concept of addition, that it is:

- **Combining two or more groups to give a total or sum**
- **Increasing an amount**

They also need to understand and work with certain principles:

- **Inverse of subtraction**
- **Commutative i.e. $5 + 3 = 3 + 5$**
- **Associative i.e. $5 + 3 + 7 = 5 + (3 + 7)$**

Adding Two Digit Numbers

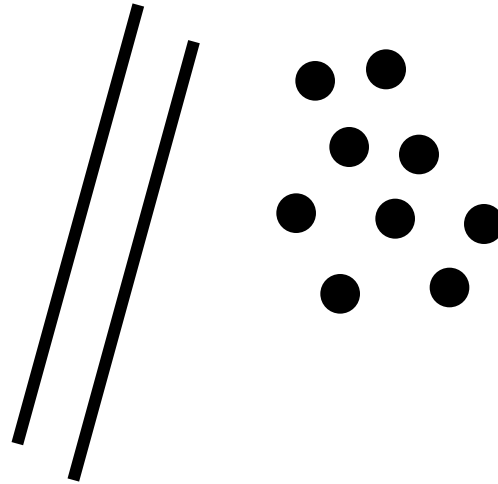
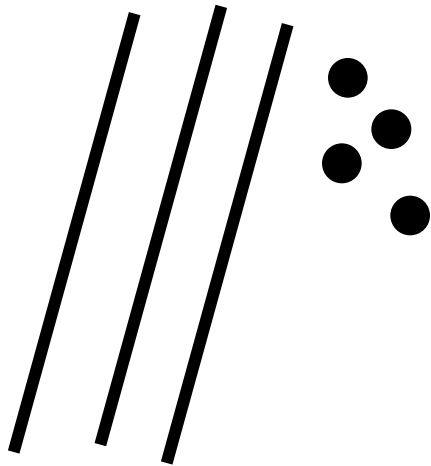


Adding two digit numbers - concrete

Children need to be able to count on in 1s and 10s from any number and be confident when crossing tens boundaries.

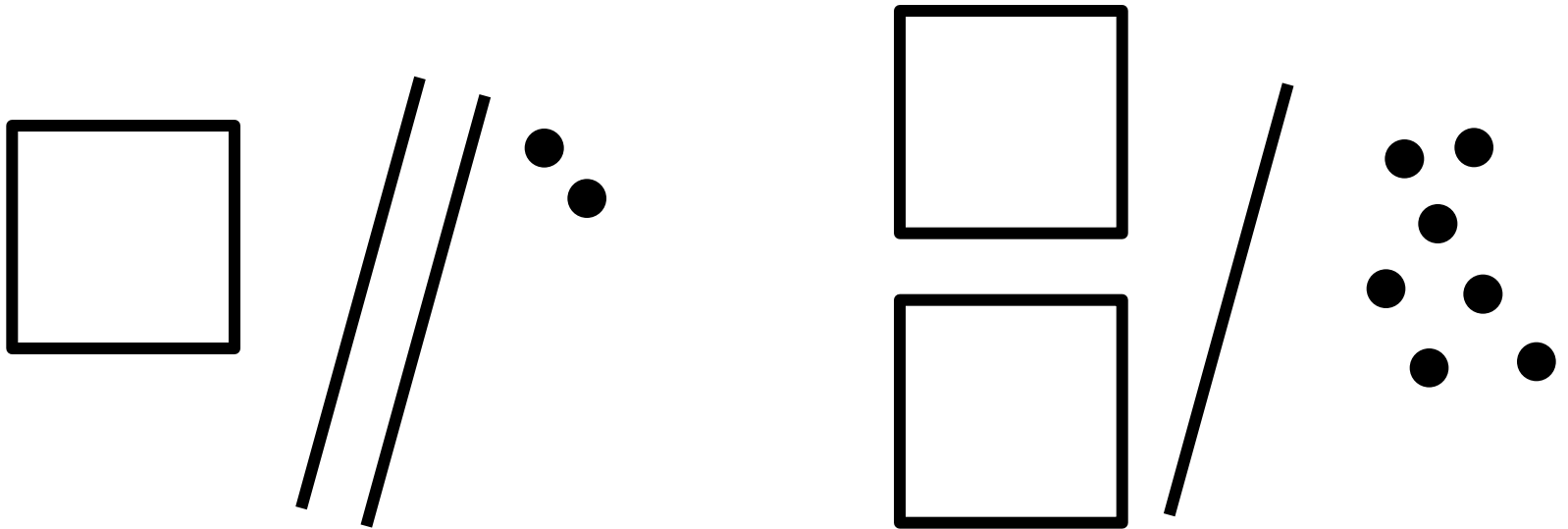
Adding Two Digit Numbers

Children can support their own calculations by using jottings,
e.g. $34 + 29$



Adding Three Digit Numbers

Children can support their own calculations by using jottings, e.g. $122 + 217$



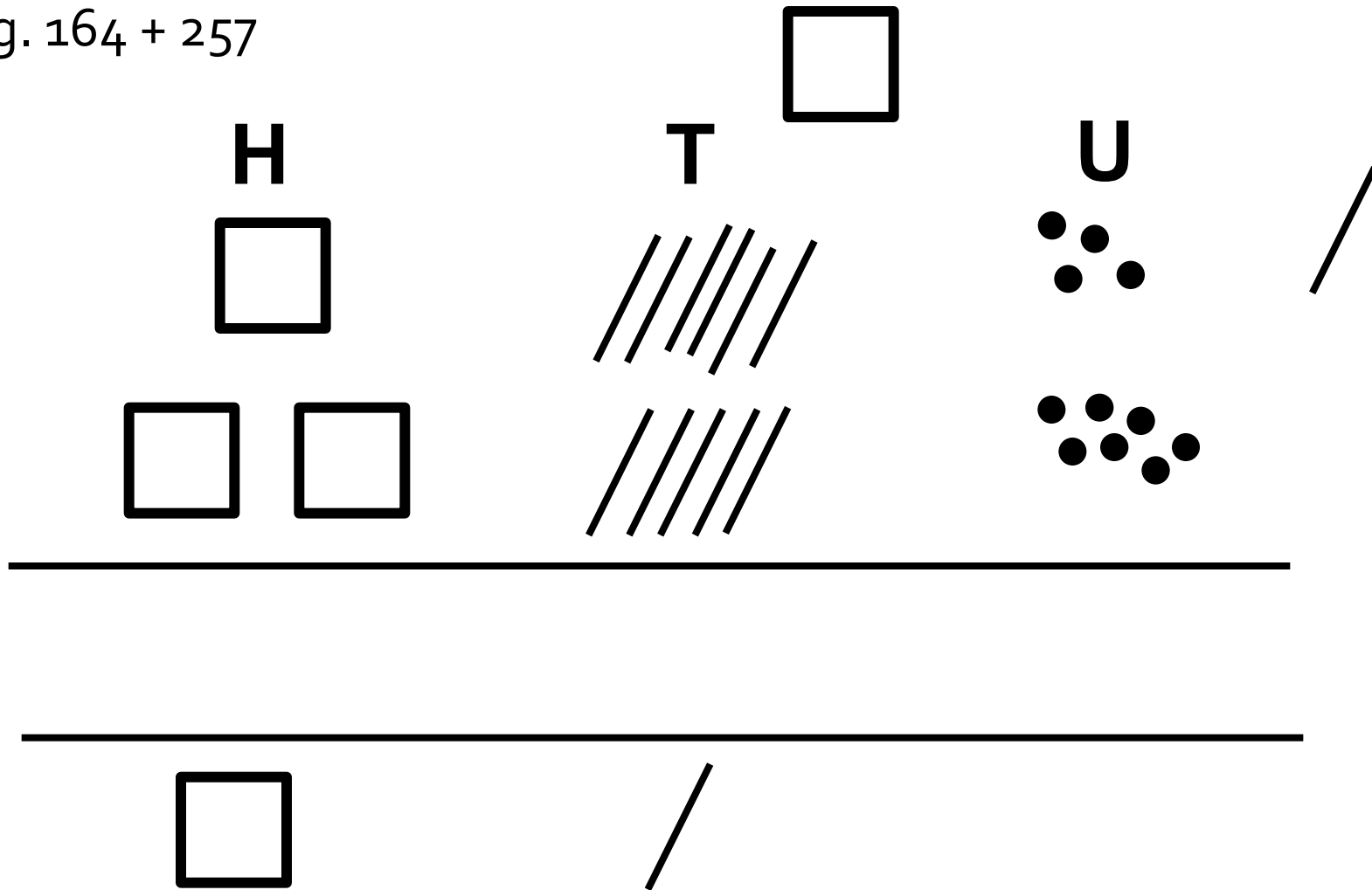
Beginning Column Addition

$$\begin{array}{r} \text{T U} \\ 67 \\ + 24 \\ \hline 11 \quad \bullet\bullet\bullet\bullet \quad \bullet\bullet \\ 80 \quad // // // // \quad // \\ \hline 91 \\ \hline \end{array}$$

The diagram illustrates the process of adding 67 and 24 using base ten blocks. The top part shows the numbers in standard form with place value labels (T for Tens, U for Units). Below this, a horizontal line separates the numbers from the intermediate steps. The next row shows the result of adding the units: 11 units, represented by two groups of ten dots (one group of four and one group of seven). The following row shows the result of adding the tens: 80, represented by four groups of ten slanted lines (one group of four and one group of four). A horizontal line is drawn below the 80. The final row shows the final sum: 91, with a horizontal line below it. The dots and slanted lines represent the base ten blocks used to model the addition.

Continuing Column Addition

e.g. $164 + 257$



Efficient Column Addition

$$\begin{array}{r} \text{H T U} \\ 166 \\ + 257 \\ \hline 423 \\ \hline 11 \end{array}$$

Subtraction – a definition

Children need to understand the concept of subtraction, that it is:

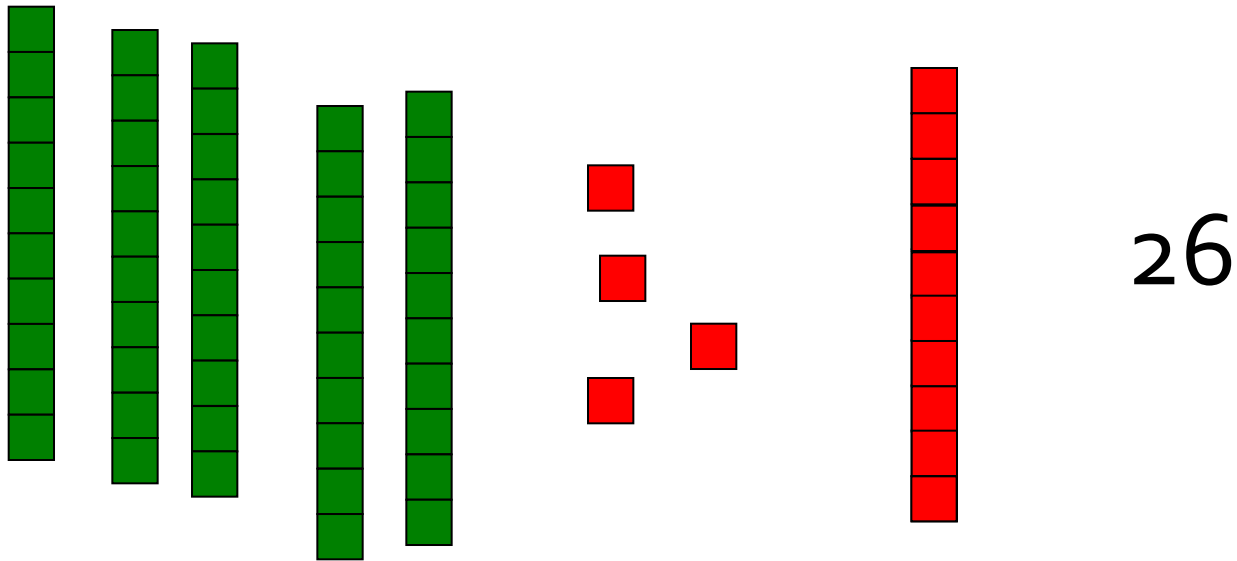
- **Removal of an amount from a larger group (take away)**
- **Comparison of two amounts (difference)**

They also need to understand and work with certain principles:

- **Inverse of addition**
- **Not commutative i.e. $5 - 3 \neq 3 - 5$**
- **Not associative i.e. $(9 - 3) - 2 \neq 9 - (3 - 2)$**

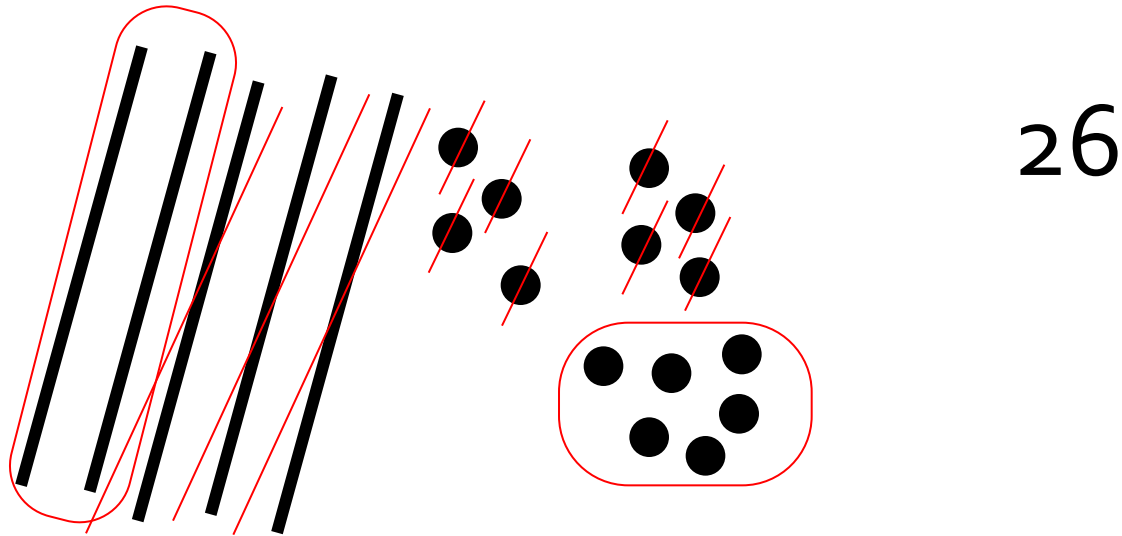
Taking Away Two Digit Numbers (Exchange)

Children can use base 10 equipment to support their subtraction strategies by basing them on counting, e.g. $54 - 28$



Taking Away Two Digit Numbers (Exchange)

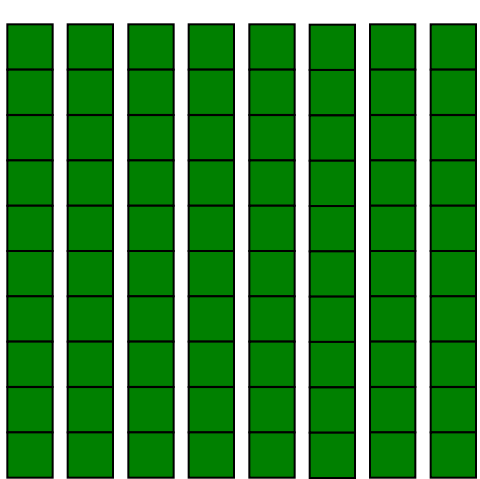
Children can support their own calculations by using jottings, e.g. $54 - 28$



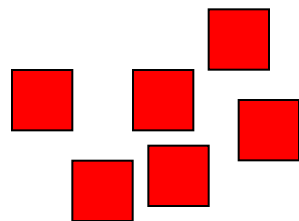
Beginning Column Subtraction

A subtraction problem is shown using arrow-shaped number cards. The top card is yellow and contains the number 86. The bottom card is yellow and contains the number 23. A minus sign is positioned to the left of the 23 card. A horizontal line is drawn below the 23 card.

$$\begin{array}{r} 86 \\ - 23 \\ \hline \end{array}$$



80



6

-

20

3

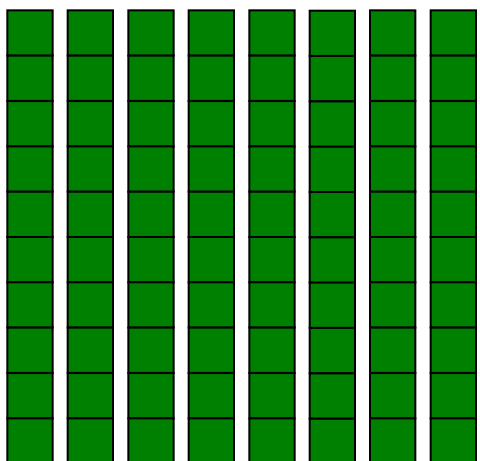


60

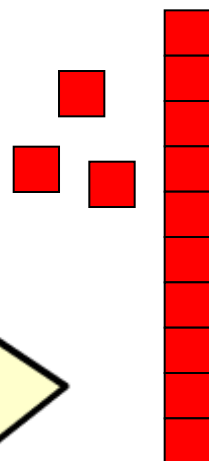
3

Beginning Column Subtraction (Exchange)

83
- 26



70



13

- 20

6

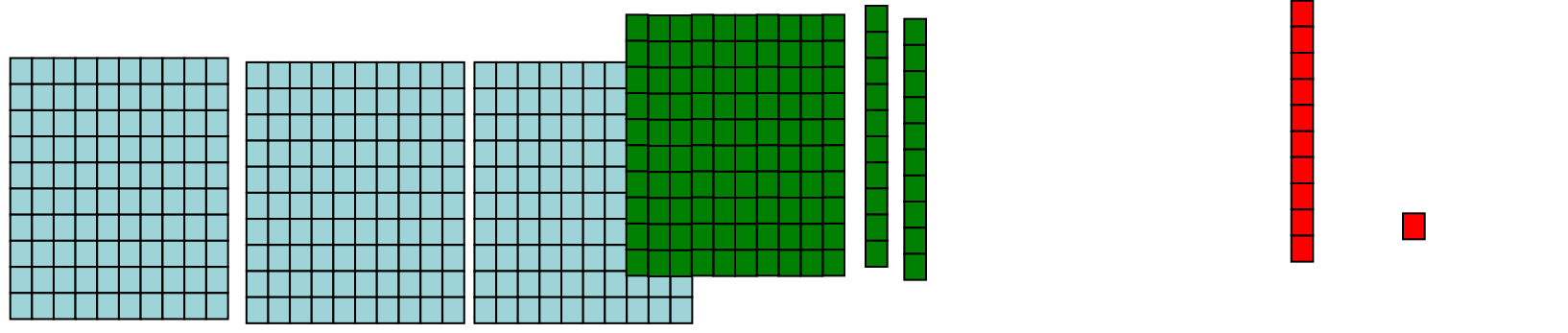


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7

Continuing Column Subtraction

e.g. 321 - 157



H	T	U	
200	110	11	
300	20	7	
- 100	50		
100	60	4	= 164

Efficient Decomposition

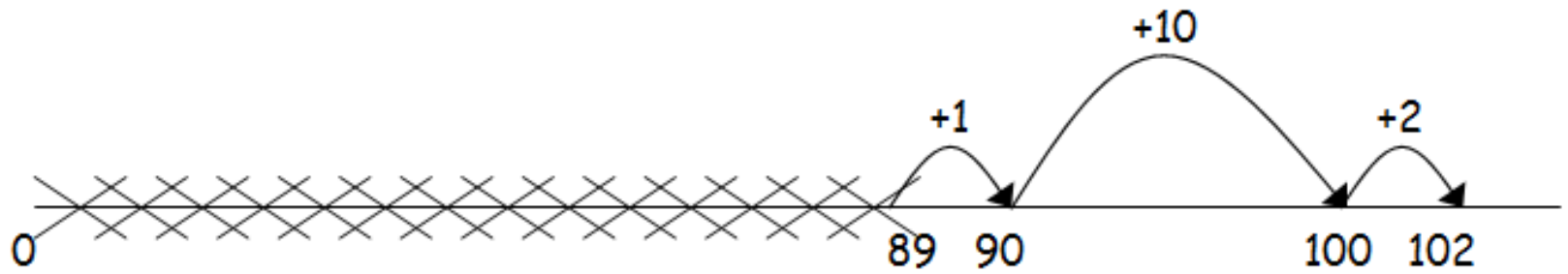
HTU

²~~3~~ ¹¹~~2~~ ¹1

+ 157

164

Using Number Lines



Multiplication

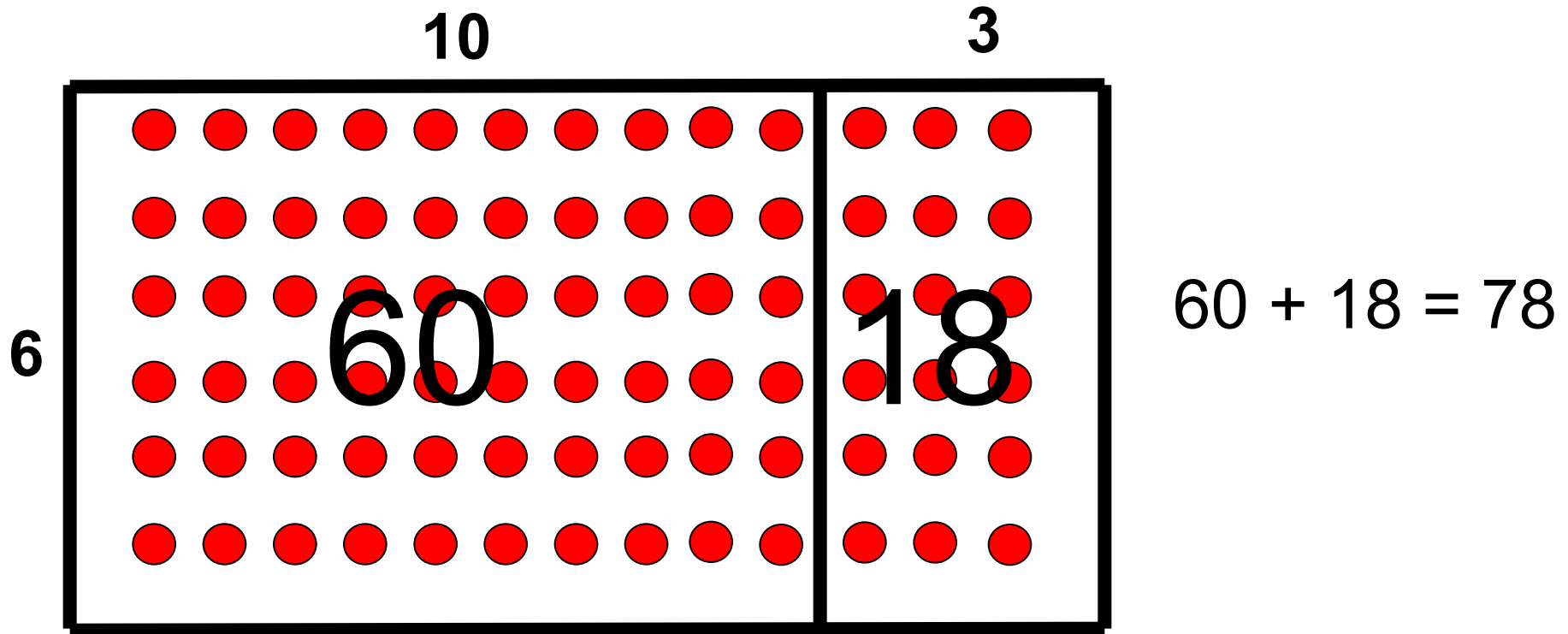
Children need to understand the concept of multiplication, that it is:

- **Repeated addition**
- **Is scaling**
- **Can be represented visually as an array**

They also need to understand and work with certain principles:

- **Inverse of division**
- **Is commutative i.e. $3 \times 5 = 5 \times 3$**
- **Is distributive i.e. $23 \times 4 = (20 \times 4) + (3 \times 4)$**
- **Is associative i.e. $2 \times (3 \times 5) = (2 \times 3) \times 5$**

Grid method of multiplication



so $13 \times 6 = 78$

Grid Method

Children have to develop their understanding of related facts.

e.g. 23×35

x	20	3
30	600	90
5	100	15

$$\begin{array}{r} 600 \\ 100 \\ 90 \\ + 15 \\ \hline 805 \\ \hline \end{array}$$

Division

Children need to understand the concept of division, that it is:

- **Repeated subtraction and it can be interpreted as sharing or grouping**

They also need to understand and work with certain principles:

- **Inverse of multiplication**
- **Is distributive i.e. $96 \div 6 = (60 \div 6) + (36 \div 6)$**
- **Is not commutative i.e. $15 \div 3 \neq 3 \div 15$**
- **Is not associative i.e. $30 \div (5 \div 2) \neq (30 \div 5) \div 2$**

$$48 \div 4 = 12$$

10 groups

2 groups



Division by Chunking

Recall of multiplication tables helps make this method more efficient, e.g. $72 \div 3$.

$$\begin{array}{r} 24 \\ 3 \overline{) 72} \\ - 30 \\ \hline 42 \\ - 30 \\ \hline 12 \\ - 6 \\ \hline 6 \\ - 6 \\ \hline 0 \end{array}$$

Answer: 24

1x	3
2x	6
5x	15
10x	30

Children should write key facts in a menu box. This will help them in identifying the largest group they can subtract in one chunk.

Division by Chunking

e.g. $196 \div 6$

$196 \div 6$

$$\begin{array}{r} \underline{32 \text{ r } 4} \\ 6 \overline{) 196} \\ - 180 \\ \hline 16 \\ - 12 \\ \hline 4 \end{array}$$



1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

The key facts in the menu box should be extended to include 4x and 20x.


Answer: 32 remainder 4 or 32 r 4

Written Division - Remainders


Remainder options:

- Keep as a whole number
- Convert to fraction
- Convert to decimal
- Round up
- Round down

Same Calculation – Different Answer

- 23 people are going out. 6 people can fit in each car.
How many cars are needed?
- Tracy has 23 vouchers. For every 6 vouchers, she gets a free CD.
How many CDs will she get?
- 6 people are sharing 23 pencils.
How can the pencils be shared out so each person has an equal number of pencils?
- 6 people are sharing 23 Swiss rolls equally. 
How many does each person get?
- 6 people won £23 between them.
How much money does each person get if shared equally?
- 6 people went out for a meal which cost £23 in total.
How much does each person need to contribute if they all contribute the same amount?
- Divide 23 by 6 on a calculator.

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Key Messages

- For written calculations it is essential that there is a progression which culminates in one method.
- The individual steps within the progression are important in scaffolding children's understanding and should not be rushed through.
- Practical equipment, models and images are crucial in supporting children's understanding.